Analyticity and Quark-Gluon Structure of Hadrons

M. Majewski*
University of Lodz, Department of Theoretical Physics
ul. Pomorska 149/153, 90-236 Lodz, Poland

V. A. Meshcheryakov†
Joint Institute for Nuclear Research
Dubna 141980, Moscow Region, Russia

February 1, 2008

Abstract

The amplitudes of hadron-hadron forward elastic scattering at high energy are investigated on the basis of analiticity and crossing-symmetry which is valid in QCD. The universal uniformizing variable for them is proposed and the formulae for crossing-even and crossing-odd amplitudes are derived. The same parameters in these formulae determine the real and imaginary (total cross sections) parts of the amplitudes. The analysis of the parameters determined from experimental data clearly points to the quark-gluon structure of hadrons. The total cross sections for hyperon-proton scattering are predicted. They are consistent with experimental data and, in particular, with the new SELEX-collaboration measurement $\sigma_{tot}(\Sigma^- p)$.

PACS numbers: 11.20.Fm, 11.50.Jg, 12.40.Aa, 13.85.Dz

1 Introduction

The modern theory of strong interactions (QCD) involves a number of unsolved problems, for instance, the problem whether glueballs exist or not. Among them, there is the problem of analytic properties of physical amplitudes. In a series of papers [1], Ohme has shown that for gauge theories quantized on the basis of BRST algebra [2] the confinement conditions can be formulated in such a way, that physical amplitudes do possess the analytic properties and conditions of crossing symmetry established earlier [3]. In particular, the forward πp

*Electronic address: mimajew@mvii.uni.lodz.pl
†Electronic address: mva@thsun1.jinr.ru

scattering amplitude has two nucleon poles and two cuts corresponding to direct and cross processes. The problem whether double dispersion relations are valid or not remains still open in gauge theories.

Below, the Ohme's result is used to construct a model of the amplitude of scattering of a hadron A on a proton. This model allows one to determine the quark-gluon structure of hadron A on the experimental basis, avoiding controversial questions on the pomeron multicomponent structure [4-6].

2 Universal Riemann surface of the forward scattering amplitude

The notion of universal Riemann surface of forword scattering amplitude for hadron-hadron processes at high energes arises when one introduces the well known variable

$$\nu = \frac{s-u}{4M\mu},$$

where s,u are usual Mandelstam varables and M,μ are the masses of colliding particles. Thresholds of any elastic hadron-hadron process corresponding to the direct and cross reactions in the s-plane transform into the points $\nu = \pm 1$. The thresholds of all inelastic processes (direc and crossing) lie on the cuts $(-\infty, -1], [+1, +\infty)$. They make the Riemann surface of scattering amplitude as function of ν infinitely-sheeted. This property of the Riemann surface can be modelled by particular choice of the uniformizing variable, the same for all hadron-hadron processes,

$$w(\nu) = 1/\pi \cdot \arcsin(\nu). \tag{1}$$

The Riemann surface of function $w(\nu)$ is just what we call the universal Riemann surface. It has three branch points: two of square root type in the points $\nu = \pm 1$ and of the logarithmic type at infinity. The function $w(\nu)$ is suitable for taking account of the crossing symmetry of amplitudes of hadron-hadron scattering. We choose the latter so that the equality

$$\mathrm{ImF}^A_\pm = \sigma^{(\bar{A}p)}_{tot} \pm \sigma^{(Ap)}_{tot}. \tag{2}$$

be valid on the upper edge of the right-hand cut of ν plane; then, the condition of crossing symmetry is:

$$\mathrm{F}_\pm(\nu) = \pm \mathrm{F}_\pm(-\nu). \tag{3}$$

Besides, the amplitudes obey the condition of reality

$$\mathrm{F}^*_\pm(\nu) = -\mathrm{F}_\pm(\nu^*). \tag{4}$$

In the w-plane, a physical sheet of the ν-plane is mapped into the strip $|\mathrm{Re}w| \leq 1/2$, whose boundaries are images of cuts of the ν-plane. We call it the physical strip in the w-plane. Nonphysical sheets of the ν-plane transform into strips

$|\mathrm{Re}(w \pm n)| \leq 1/2, (n = 1, 2, \cdots)$. This clearly demonstrates that the universal Riemann surface is infinite-sheeted.

Let $w = x + iy$. Then, owing to eq. (3-4), on the boundary of the physical strip we find

$$\mathrm{F}^*_{\pm}(1/2 + iy) = \mp \mathrm{F}_{\pm}(-1/2 + iy). \tag{5}$$

Let us expand the amplitudes $\mathrm{F}_{\pm}(w)$ into Taylor series with the center at the point $w_0 = iy_0$. It's convergence radius is determined by the distance from the point w_0 to the nearest pole corresponding to the resonance on an unphysical sheet. The same parameters of that expansion determine both the real and imaginary parts of amplitudes $\mathrm{F}_{\pm}(w)$. Below, we will use only the imaginary parts of amplitudes (the total cross sections) that can be represented by the following converging power series

$$\mathrm{I}m\mathrm{F}_{+}(1/2 + iy) = \sum_{n \geq 1} \left(\frac{1}{2}\right)^{2n-2} \sigma_{+}^{(n)}(y),$$

$$\sigma_{+}^{(1)}(y) = \sum_{m \geq 1} a_m (y - y_0)^{m-1}, \qquad \sigma_{+}^{(n)}(y) = \frac{(-1)^{n+1}}{(2n-2)!} \cdot \frac{d^{2n-2}\sigma_{+}^{(1)}(y)}{dy^{2n-2}},$$

$$\mathrm{I}m\mathrm{F}_{-}(1/2 + iy) = \sum_{n \geq 1} \left(\frac{1}{2}\right)^{2n-1} \sigma_{-}^{(n)}(y), \tag{6}$$

$$\sigma_{-}^{(1)}(y) = \sum_{m \geq 1} b_m (y - y_0)^{m-1}, \qquad \sigma_{-}^{(n)}(y) = \frac{(-1)^{n+1}}{(2n-2)!} \cdot \frac{d^{2n-2}\sigma_{-}^{(1)}(y)}{dy^{2n-2}}.$$

Expansions (6) satisfy equation (5). It is instructive to compare the argument of expansions (6) with commonly used expressions, for instance: $(\frac{s}{s_0})^{\alpha}, s_0 = 1\mathrm{GeV}^2$ in refs. [4,6,8] and $(p/20)^{\alpha}$ in ref. [9] (here and everewere below p is the momentum in the lab. system). However, when one attempts to compare two different parametrizations of total cross sections in the region ~ 100 GeV/c, the function $\ln(p/p_0)$ arises naturally. Let us derive it from formula (6). From (1) it follows that $y = \ln(\nu + \sqrt{\nu^2 - 1})$. For $s \gg M^2$, we have $y \sim \ln(2p/\mu)$. In this case, the function $(y - y_0) \sim 1/\pi \ln(p/p_0)$ is the argument of expansions (6). Here the quantity p_0 has clear mathematical meaning— it is the center of the expansion into the Taylor series, and at the same time, physically, it makes p dimensionless. We stress once more that formulae (6) are valid in the vicinity of point y_0, and they cannot be used to estimate behavior of cross sections when $s \to \infty$; discussions on the pomeron structure refer to the region where they are not applicable.

3 Quark-gluon structure of hadrons

Formulae (6) were employed to analyze the experimental data on total cross sections pp, $\bar{p}p, K^{\pm}p, \pi^{\pm}p$. [7]. The results are collected in the Table I. Twenty four coefficients a_m, b_m are determined by 300 experimental points and describe

Table 1: The values of the parameters a_m, b_m(all in mb) and y_0 in eq. (6).

	pp	πp	kp	np
a_1	84.51 ± 0.18	49.77 ± 0.09	41.03 ± 0.12	83.49 ± 0.36
a_2	-4.85 ± 0.36	1.92 ± 0.19	5.16 ± 0.25	-3.48 ± 0.62
a_3	15.97 ± 0.7	10.37 ± 0.34	7.37 ± 0.48	8.72 ± 1.48
b_1	8.52 ± 0.17	1.62 ± 0.07	3.51 ± 0.12	7.85 ± 0.26
b_2	-13.82 ± 0.79	-2.8 ± 0.17	-5.65 ± 0.51	-12.74 ± 1.24
b_3	15.33 ± 1.7	2.7 ± 1.8	5.04 ± 0.97	12.36 ± 2.97
y_0	1.71	2.31	1.91	1.71
$\chi^2_{n_D}$	$\frac{112}{109}$	$\frac{82}{73}$	$\frac{48}{38}$	$\frac{96}{50}$

behaviour of cross sections in the interval $p \in (10, 10^3)\,\mathrm{GeV}/c$. Values of y_0 correspond to $p = 100\,\mathrm{GeV}/c$, at which the correlations between parameters a_m, b_m are minimal. In the vicinity of y_0, the considered total cross sections have minima, and the real parts of amplitudes cross over the zero. Twelve coefficients b_m display the simple dependence:

$$(b_m)_{pp} : (b_m)_{\pi p} : (b_m)_{Kp} : (b_m)_{np} = 5 : 1 : 2 : 4.$$

The mean ratios are calculated from Table 1 to be as follows:

$$\overline{\left(\frac{b_{pp}}{b_{\pi p}}\right)} = 5.37 \pm 0.22 \qquad \overline{\left(\frac{b_{Kp}}{b_{\pi p}}\right)} = 2.16 \pm 0.12 \qquad \overline{\left(\frac{b_{np}}{b_{\pi p}}\right)} = 4.79 \pm 0.23.$$

They are in good agreement with ratios (7), except for the last one. It differs from (7) by three standard deviations as a result of large χ^2/n_D for np scattering.Therefore, it is expedient to use it below only for qualitative estimations. Relations (7) are not new and are written in order to demonstrate that the analysis of coefficients a_m, b_m is important for determining quark and other degrees of freedom of hadrons. It is known [9], that relationships (7) follow from consideration of annihilation components of amplitudes and are proportional to the numbers of dual diagrams of scattering of a hadron on proton

$$n_d(Ap) = 2N^A_{\bar{u}} + N^A_{\bar{d}} \tag{7}$$

where $N^A_{\bar{u}}, N^A_{\bar{d}}$ are numbers of antiquarks $\bar{u}, \bar{d}$ in hadron A.

It is of great interest but difficult to analyze the crossing even part of the scattering amplitude. The additive quark model (AQM) [10] predicts the following ratios

$$\sigma_{pp} : \sigma_{\pi p} : \sigma_{Kp} : \sigma_{np} = 3 : 2 : 2 : 3$$

However, from our Table 1 it is seen that only the coefficients a_1 and a_3 approximately follow that dependence. The difference $(a_1)_{pp} - (a_1)_{np} = 1.02 \pm 0.40$ can be considered compatible with zero, since it does not exceed three standard deviations, and the description of process *np* is not quite satisfactory. We will

neglect the distinction between processes pp and np, though, for the coefficient a_3, this assumption is valid only due to χ^2/n_D being large in magnitude. At the same time, the difference $(a_1)_{\pi p}-(a_1)_{kp} = 8.74\pm0.15$ is significant and, together with other coefficients, determines 30 % accuracy of the additive quark model. The values of coefficients a_2 from Table 1 do not comport with the AQM predictions, and therefore, they are very important for choosing new models. Some attempts of refining the AQM are known [11, 12]. All of them suggest that the amplitude should be supplemented with terms bilinear in quark numbers of hadrons A. In this case, the amplitude can be described satisfactorily under different assumptions on the form of bilinear terms. However, their clear physical justification is rather difficult.

Below, we construct a new model by using the known idea of quarks being confined in a hadron by gluons. Then it is natural to assume that the total cross section of scattering of hadron A on a proton contains a part that describes gluon-gluon interaction. With this in mind, we set

$$a_m = \alpha_m + \beta_m \cdot N_q^A + \gamma_m \cdot N_q^A \cdot N_{ns}^A \tag{8}$$

where N_q^A is the total number of quarks; N_{ns}^A is the total number of nonstrange quarks in hadron A; and the numbers α_m do not depend on the quark content of hadron A [9]. The numbers α_m determine the fraction of the total cross section corresponding to the gluon-gluon interaction. It is just the gluon degree of freedom of hadrons A and p that is responsible for them. The assumption on a_m (eq.(8)) corresponds to the hypothesis of Gershtein and Logunov [13]. They argue that the constant of Froissart limit doesn't depend on the guark content of hadron A , but it does depend on glueballs and is the same for all processes. The hypothesis has been verified by Prokoshkin [14] on the basis of similar experimetal data as we use. In our model one should attribute the Froissart behavior not to the variable y but to y_0 one. That justifies the eq.(8). The different numbers $(a_m)_{pp}, (a_m)_{\pi p}, (a_m)_{kp}$ determine $\alpha_m, \beta_m, \gamma_m$. Then, the prediction power of hypothesis (8) can be verified for the values of total cross sections of hyperon-proton interactions. In ref. [15], the results are presented on the measurement of total cross sections of $\Sigma^- p$ and $\Xi^- p$ in the range of momenta $(74.5, 136.9)\,\mathrm{GeV}/c$. In this range, the total cross sections are varying slightly, and to compare the predictions of the model given by formulae (1), (6), and (8), we take the momentum $p = 101\,\mathrm{GeV}/c$. In this case, the theoretical and experimental results are as follows:

$$\sigma_{\Xi^- p} = \begin{array}{l} (29.25 \pm 0.5\,\mathrm{mb})_{th} \\ (29.12 \pm 0.22\,\mathrm{mb})_{ex} \end{array}, \quad \sigma_{\Sigma^- p} = \begin{array}{l} (34.8 \pm 0.2\,\mathrm{mb})_{th} \\ (33.3 \pm 0.3\,\mathrm{mb})_{ex} \end{array}$$

Similar data [16] for Λp and $\Sigma^- p$ scattering at $20\,\mathrm{GeV}/c$ are

$$\sigma_{\Lambda p} = \begin{array}{l} (33.3 \pm 0.5\,\mathrm{mb})_{th} \\ (34.7 \pm 3\,\mathrm{mb})_{ex} \end{array}, \quad \sigma_{\Sigma^- p} = \begin{array}{l} (34.2 \pm 0.5\,\mathrm{mb})_{th} \\ (34 \pm 1\,\mathrm{mb})_{ex} \end{array}$$

Recently, the collaboration SELEX has published the data on $\Sigma^- p$ at $p =$

609 GeV/c. [17]. The comparison with predictions of the model is

$$\sigma_{\Sigma^- p} = \begin{array}{l} (35 \pm 7.5\,\mathrm{mb})_{th} \\ (37 \pm 0.7\,\mathrm{mb})_{ex} \end{array}$$

Though the obtained value of the total cross section is not so accurate as in ref. [18], it should be considered satisfactory. In refs. [4, 5, 6, 11, 18] devoted to the analysis of total cross sections, the errors of predicted values were not calculated, but they increase rapidly in the region of extrapolation.

4 Conclusion

A uniformizing variable for hadron-hadron forward scattering at high energes was proposed on the basis of analyzing the analytic properties of physical scattering amplitudes [1]. If one represents the scattering amplitudes as Taylor series in that variable and takes crossing symmetry into account, one can once more be convinced on the experimental basis that hadrons possess the quark-gluon structure. Predicted the total cross sections for scattering of strange hadrons on proton are in agreement with experiment in a wide energy range. The gluon-gluon part of the total cross sections at momenta $p = 100\,\mathrm{GeV}/c$ amounts to about 10%.

References

[1] R. Oehme, Int. J. of Mod. Phys. **A10**, 1995 (1995) and references therein.

[2] C. Becchi, A. Rouet, R. Stora, Ann. Phys. **98**, 2879 (1976);
I.V. Tyutin, Lebedev Report , FIAN 39 (1975).

[3] N.N. Bogoliubov, B.V. Medvedev, M.K. Polivanov,
Voprosy Teorii Dispersionnykh Sootnoshenii, Fizmatgiz, Moscow, 1958.

[4] A. Donnachie, P.V. Landshoff, Phys. Lett. **B296**, 227 (1992).

[5] H.J. Lipkin, Phys. Lett. **B335**, 500 (1994).

[6] P. Gauron, B. Nicolescu, A possible two-component structure of the non-perturbative Pomeron, preprint LPNE 00-02, Paris.

[7] V.P. Gerdt, V.I. Inozemtsev, V.A. Meshcheryakov,
Lett. Nuovo Cimento **15**, 321 (1976).

[8] Particl Data Grup, Europian Physics Jornal **C3**, 205 (1998).

[9] H.J. Lipkin, Phys. Rev. **D11**, 827 (1975].

[10] E.M. Levin, L.L. Frankfurt, Pisma JETP bf 2, 105 (1965).

[11] P. Jonson, B. Nicolescu, Nuovo Cim. **A37**, 97 (1977).

[12] H.J. Lipkin, Nucl. Phys. **B78**, 1381 (1974).

[13] S.S. Gershtein, A.A. Logunov, Yad. Fiz. **39**, 1514 (1984).

[14] Y.D. Prokoshkin Yad. Fiz. **40**, 1579 (1984).

[15] S. Gjesdal et al., Phys. Lett. **B40**, 152 (1972).

[16] S.F. Biagis et al., Nucl. Phys. **B186**, 1 (1981).

[17] U.Dersch et al., Nucl. Phys. **B579**, 277 (2000).

[18] H.J. Lipkin, The New $\sigma_{tot}(\Sigma p)$ data, the new PDG fit to hadron total cross section and the TCP alternative, hep-ph/9911259.